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Engineering Notes

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Unified Approach for Roll Ratcheting Analysis

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Introduction

THE introduction of digital fly-by-wire flight control systems has increased the potential for adverse interactions between the human pilot and the aircraft dynamics. In the past these phenomena have been called pilot-induced oscillations (PIOs). However, this expression blames the pilot, whereas it is generally accepted that PIO is no pilot failure. Therefore, it was recommended to use the expression aircraft-pilot coupling (APC).[1]

The cause for APC is characterized by a rich variety of highly diverse phenomena in terms of effective aircraft dynamics and pilot behavioral modes.[2] For the purpose of this Note APCs can be divided into two basic frequency regimes. Dangerous APCs occur at frequencies around the pilot's crossover frequency up to a frequency of 10 rad/s (Ref. 3). In the roll axis high-frequency, low-amplitude oscillations are known as roll ratcheting (RR). These oscillations basically occur at frequencies above 10 rad/s and are caused by interactions between the cockpit controls and the pilot's neuromuscular system.[4]

A great database has been generated in the Lateral High Order System (LATHOS) program to support handling qualities investigations of fighter aircraft in the roll axis.[5] This flight test program also includes data and an analysis of RR. It has been shown that a low roll mode time constant increases the potential for RR. Closed-loop models of pilot-aircraft systems are suitable tools for RR investigations. An early approach was made by using a modification of the crossover model that indicated closed-loop pilot-aircraft oscillations of 2 to 3 Hz (Ref. 6). However, RR may also occur in large transport aircraft because of aeroelastic effects such as those observed with a C-17 aircraft.[7]

In this Note a unified approach for RR analysis is presented. A new four-body manipulation model including the control stick, the pilot's arm, torso, and hip is used within a closed-loop pilot-aircraft system. The model is successfully applied to an RR incident of the F-16 XL. A new RR prediction criterion based on stability margins in the frequency domain is presented.

Background

Two manipulation models containing the biomechanical dynamics of the pilot body and the stick characteristics were developed and validated in former laboratory experiments.[8] One model included the dynamics of four bodies, the stick, the arm, the torso, and the hip, whereas a second model consisted of two bodies, the stick and the arm. To validate the models special experiments were performed with a pilot seat, a stick, and a monitor on a platform. During a tracking task the bank angle was displayed on the monitor while the platform was accelerated laterally. The stick deflection and the lateral acceleration of the pilot's shoulder were measured. Experiments with and without lateral accelerations utilizing a stiff and a springy stick were conducted. In the tracking tests without lateral accelerations pilot models of the compensatory tracking behavior were determined. In tracking tests with lateral accelerations the compensatory tracking behavior of the pilot was assumed to be unchanged and the dynamics of the manipulation system were identified.

The two-body manipulation model with the stick and the arm was modified and then used to reproduce the RR incident of the F-16 XL.[9] In that case a copilot in the F-16 XL had noticed severe RR particularly at high commanded roll rates and during recovery (Fig. 1). The analysis of that RR incident is also the subject of this Note with the following two major objectives: 1) reproduction of the RR with a more realistic manipulation model and 2) development of a RR criterion based on a manipulation model.

Four-Body Manipulation Model

A new four-body manipulation model representing the dynamics of the stick, arm, torso, and hip has been developed regarding the following aspects[10]:

1) The system response because of the lateral and roll acceleration is considered in the model structure.

2) The model parameters have been identified simultaneously from the test data of the stick force and shoulder acceleration.

3) A third-order Padè approximation was used to represent the pilot's time delay.

The frequency response amplitudes of the identified manipulation model are represented in Fig. 2. The test data of experiments with a stiff and a springy stick are well matched within the frequency range considered.

F-16 XL Roll Ratcheting

The RR incident of the F-16 XL has been analyzed using the new four-body manipulation model with the aircraft and flight control system dynamics.[10] First a stability analysis in the frequency

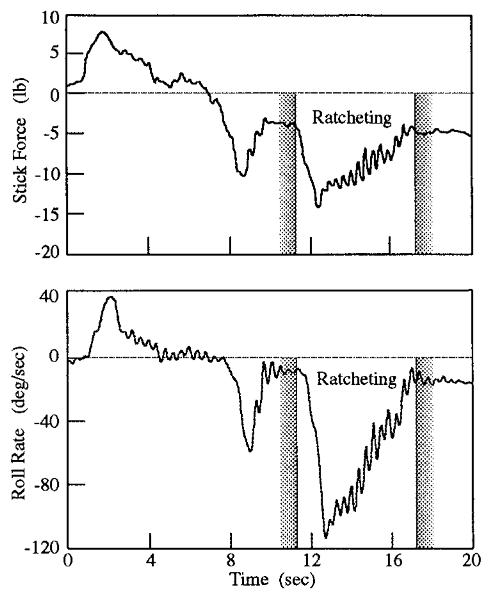

Fig. 1 F-16 XL time histories during rolling maneuvers.[9]

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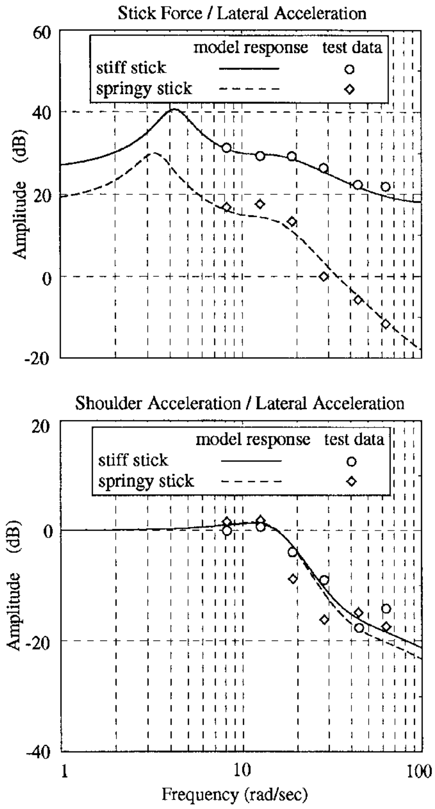

Fig. 2 Amplitudes of stick force and shoulder acceleration.

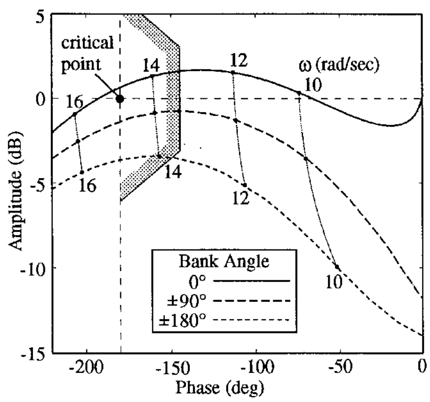

Fig. 3 Effect of bank angle changes (F-16 XL).

domain utilizing linearized models was performed. Second, a nonlinear simulation of a roll maneuver was conducted.

Stability Analysis

Nichols charts are commonly used tools to assess the stability of feedback systems. The open-loop frequency response of the pilot-aircraft system in the F-16 XL has been calculated for different bank angles. The representation in Fig. 3 shows that the closed-loop system is unstable for a zero bank angle as the curve is above the critical point. However, it becomes slightly stable for increasing bank angles. Stability margins of flight control laws defined in design and clearance requirements are included in the diagram. Though the requirement is not designed for RR analysis, these boundaries can be utilized for RR analysis as well. A comparison of the frequency response with the boundaries shows that the function violates the requirement for all bank angles in a rolling maneuver.

The experience from flight tests has shown that most extensive RR occurred during recovery (Fig. 1). This effect can be explained by a nonlinear dual lag filter in the flight control system. To analyze this effect in the frequency domain three describing functions were developed with regard to positive, zero, and negative commanded roll accelerations.[10] Figure 4 shows the dependence of the stability on the commanded roll acceleration for zero bank angle. During decreasing roll commands the model is more unstable than in constant rolls, as has been observed in flight tests.

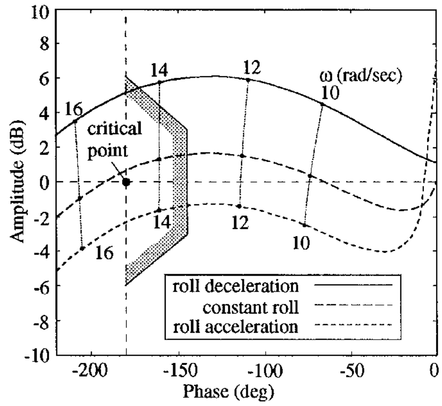

Fig. 4 Effect of the dual lag filter (F-16 XL).

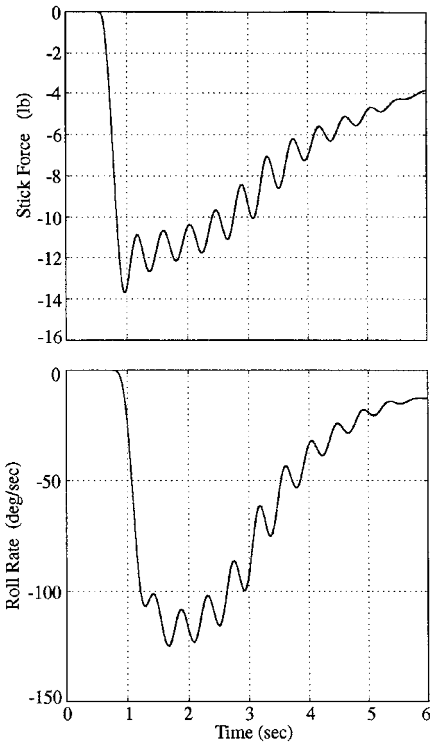

Fig. 5 Nonlinear simulation (F-16 XL).

Nonlinear Simulation

Finally a nonlinear simulation of the F-16 XL pilot-aircraft loop has been conducted to verify the results from stability analysis. The input signal to the closed-loop pilot-aircraft model is a pilot force in the shoulder muscles intended to move the stick. Simulation results of a F-16 XL roll maneuver are shown in Fig. 5. Significant oscillations of the pilot-aircraft system occur. The amplitudes and frequency of the oscillations are consistent with the flight test results presented in Fig. 1. The oscillations do not decay until the stick force decreases below a certain value. Then a lower gain in the flight control system results in an increased stability and the oscillations fade off.

Conclusion

A new mathematical model to simulate the biomechanical dynamics of the pilot body and the stick characteristics has been developed. The new manipulation model has been successfully used to analyze the roll ratcheting incident of an F-16 XL.

Nonlinear effects because of large bank angle changes and the response characteristics of the dual lag filter have been investigated in the frequency domain. Nonlinear simulations in the time domain provided very similar results as obtained during flight tests.

A new frequency domain criterion for RR prediction is proposed based on stability margins in a Nichols chart. The proposed procedure should be further developed to obtain a well-defined RR criterion for flight control system designers applicable to highly maneuverable fighters and to large flexible transport aircraft.

References

[1]McKay, K., "Summary of an AGARD Workshop on Pilot-Induced Oscillations," AIAA Paper 94-3668, Aug. 1994.

[2]McRuer, D. T., Droste, C. S., Hansman, R. J., Hess, R. A., LeMaster, D. P., Matthews, S., McDonnel, J. D., McWha, J., Melvin, W. W., and Pew, R. W., *Aviation Safety and Pilot Control: Understanding and Preventing Unfavorable Pilot-Vehicle Interactions*, National Academy, Washington, DC, 1997.

[3]McRuer, D., "Pilot Induced Oscillations and Human Dynamic Behavior," NASA CR 4683, 1995.

[4]Ashkenas, I. L., "Pilot Modeling Applications," AGARD LS-157, Paper 3, Delft, Netherlands, 1988.

[5]Monagan, S. J., Smith, R. E., and Bailey, R. E., "Lateral Flying Qualities of Highly Augmented Fighter Aircraft," Air Force Wright Aeronautical Labs, TR-81-3171, Vol. 1, Wright–Patterson AFB, OH, March 1982.

[6]Chalk, C. R., "Excessive Roll Damping Can Cause Roll Ratchet," *Journal of Guidance, Control, and Dynamics*, Vol. 6, No. 3, 1983, pp. 218, 219.

[7]Norton, W. J., "Aeroelastic Pilot-in-the-Loop Oscillations," AGARD-AR-335, Neuilly-Sur-Seine, France, Feb. 1995.

[8]Allen, R. W., Jex, H. R., and Magdaleno, R. E., "Manual Control Performance and Dynamic Response During Sinusoidal Vibration," Systems Technology, Inc. TR-1013-2, Hawthorne, CA, Oct. 1973.

[9]Smith, J. W., and Montgomery, T., "Biomechanically Induced and Controller Coupled Oscillations Experienced on the F-16 XL Aircraft During Rolling Maneuvers," NASA TM 4752, July 1996.

[10]Koehler, R., "A New Model for Roll Ratcheting Analysis," Institut für Flugmechanik, IB 111-97/32, DLR Braunschweig, Germany, 1997.

Linear Quadratic Optimality of Infinite Gain Margin Controllers

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Introduction

INFINITE gain margin controllers (IGMCs) are a class of robust controllers designed with guaranteed stability in the presence of a gain uncertainty factor. The loop transfer functions recovered by them are generally minimum phase with no right half-plane zeros.[1,2] Although seeking robustness, it is also known that the IGMCs deteriorate nominal performance, and so the associated linear quadratic (LQ) cost functions are not necessarily minimum. We consider the cost functions for single input LQ regulators and investigate optimality of the controllers seeking IGM. The problem is stated as follows: Given $\boldsymbol{k}$, an IGMC and a scalar $\rho \geq 1$, does there exist a $\rho \in [1, \infty)$ for which $\rho\boldsymbol{k}$ is LQ optimal? In output feedback setting, this problem is a simple extension to the inverse problem of state feedback LQ regulators.[3] Both frequency- and time-domain optimality criteria are considered for analysis. Similar results for other classes of controllers and cost functions are addressed in the literature.[4,5]

Background

Consider a linear time-invariant n-state single input dynamic system,

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}u(t) \tag{1}$$

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where $\boldsymbol{x}(t)$ and $u(t)$ are the state and control input signals. $\boldsymbol{A}$ and $\boldsymbol{b}$ are real compatible matrices. Whenever $x(0) = x_0$ is nonzero, it is known that the control law $u(t)$ uses a constant gain row vector $\bar{\boldsymbol{k}}$, linear to sensor outputs $\boldsymbol{z}(t) \in \boldsymbol{R}^r$, and regulates desired outputs $\boldsymbol{y}(t) \in \boldsymbol{R}^p$ to a set point (zero, assumed). Using $\boldsymbol{C} \in \boldsymbol{R}^{r \times n}$ and $\boldsymbol{H} \in \boldsymbol{R}^{p \times n}$, we write $\boldsymbol{z}(t)$, $u(t)$, and $\boldsymbol{y}(t)$ as

$$\boldsymbol{z}(t) = \boldsymbol{C}\boldsymbol{x}(t) \tag{2}$$

$$u(t) = -\bar{\boldsymbol{k}}\boldsymbol{z}(t) \tag{3}$$

$$\boldsymbol{y}(t) = \boldsymbol{H}\boldsymbol{x}(t) \qquad \boldsymbol{x}_0 \neq 0 \tag{4}$$

Within this setting, recall the inverse problem of LQ regulators for $\boldsymbol{z}(t) = \boldsymbol{x}(t)$ proposed by Kalman[3] using a cost function J,

$$J = \int_0^\infty \{\boldsymbol{y}'\boldsymbol{y} + u^2\}\,dt = \int_0^\infty \{\boldsymbol{x}'\boldsymbol{H}'\boldsymbol{H}\boldsymbol{x} + u^2\}\,dt \tag{5}$$

$\boldsymbol{V}'$ is the transpose of a matrix or a vector $\boldsymbol{V}$. Note that the regulation (or performance) problem considered by Kalman has two categories. In the first case, $\boldsymbol{H}$ is fixed and so is the positive semidefinite (PSD) matrix $\boldsymbol{Q} = \boldsymbol{H}'\boldsymbol{H}$ in J (see Theorem 5 in Ref. 3 for optimality conditions). In the second case, J assumes several $\boldsymbol{H}$ for which a given state feedback control law may be optimal (Theorem 6 in Ref. 3, inverse problem of linear optimal control theory). When $\boldsymbol{y}(t)$ in J is unspecified, the algebraic optimality criteria (Theorem 4 in Ref. 3) determine all possible $\boldsymbol{H}$ for which J is possibly minimum.

In this paper, the inverse problem of linear optimal control theory is presented for controllers seeking IGM. That is, for a stabilizing output feedback gain $\rho\boldsymbol{k}$ with $\rho \geq 1$ in the control law given by

$$u(t) = -\rho\boldsymbol{k}\boldsymbol{x}(t) \tag{6}$$

the problem is to find whether or not $u(t)$ is optimal for any $\rho \in [1, \infty)$. Whenever necessary, $\boldsymbol{k}_x = \bar{\boldsymbol{k}}\boldsymbol{I}_n$, with identity matrix $\boldsymbol{I}_n$ in $\boldsymbol{k} = \bar{\boldsymbol{k}}\boldsymbol{C}$, is used for specializing the state feedback results.[3] The implications of complete controllability and observability of the pairs $[\boldsymbol{A}, \boldsymbol{b}]$ and $[\boldsymbol{k}, \boldsymbol{A}]$ are discussed in Ref. 3. In addition, the following assumptions hold for the output feedback case.

Assumption 1: All nonzero initial conditions $\boldsymbol{x}_0$ satisfying the expected value $E(\boldsymbol{x}_0'\boldsymbol{x}_0) = \boldsymbol{X}$, where $\boldsymbol{X}$ is a set of all positive definite (PD) symmetric matrices, are observable. Thus, $[\boldsymbol{C}, \boldsymbol{A}]$ is completely observable.

Assumption 2: The pair $[\boldsymbol{H}, \boldsymbol{A}]$ is completely observable. Unless $\boldsymbol{x}_0$ in $\boldsymbol{X}$ is zero, this assumption excludes the possibility of $\boldsymbol{y}(t)$ reaching the set point without $u(t)$. This may happen when $\boldsymbol{A}$ is asymptotically stable.

Following the assumptions, the algebraic optimality criteria for $\boldsymbol{k}$ to be LQ optimal are[6]

$$\boldsymbol{P} = \boldsymbol{P}' \text{ (PD)} \tag{7}$$

$$(\boldsymbol{A} - \boldsymbol{b}\boldsymbol{k})'\boldsymbol{P} + \boldsymbol{P}(\boldsymbol{A} - \boldsymbol{b}\boldsymbol{k}) = -(\boldsymbol{Q} + \boldsymbol{k}'\boldsymbol{k}) \tag{8}$$

$$(\boldsymbol{A} - \boldsymbol{b}\boldsymbol{k})'\boldsymbol{S} + \boldsymbol{S}(\boldsymbol{A} - \boldsymbol{b}\boldsymbol{k}) = \boldsymbol{X} \tag{9}$$

$$\bar{\boldsymbol{k}} = \boldsymbol{b}'\boldsymbol{P}\boldsymbol{S}\boldsymbol{C}'(\boldsymbol{C}\boldsymbol{S}\boldsymbol{C}')^{-1} \tag{10}$$

Note that $\boldsymbol{P}$ in Eq. (8) is not unique unless $\boldsymbol{Q} + \boldsymbol{k}'\boldsymbol{k}$ is PSD.[7] The controller $\bar{\boldsymbol{k}}$ in Eq. (10) is optimal for the cost function

$$J = E\left\{\int_0^\infty (\boldsymbol{x}'\boldsymbol{Q}\boldsymbol{x} + u^2)\,dt\right\} \tag{11}$$

When $\boldsymbol{C} = \boldsymbol{I}_n$, the state feedback gain is $\boldsymbol{k}_x = \boldsymbol{b}'\boldsymbol{P}\boldsymbol{S}\boldsymbol{S}^{-1}$, and Eqs. (8) and (9) are equivalent. Thus, the algebraic optimality conditions simplify to the results of Kalman,[3]

$$\boldsymbol{P} = \boldsymbol{P}' \text{ (unique and PD)} \tag{12}$$